Find the 5^{th} term in the expansion of $(5n^2 - 7n^3)^{26}$. SCORE: _____ / 5 PTS

NOTES: **Your coefficient may use the operations +, –, × and positive exponents only.**
All exponents must be simplified and all signs must be explicit (at the front).

$${}_{26}C_4 (5n^2)^{26-4}(-7n^3)^4 = {}_{26}C_4 (5n^2)^{22}(-7n^3)^4$$

$$= \frac{26!}{4!22!} 5^{22} n^{44} (-7)^4 n^{12}$$

$$= \boxed{\frac{26 \cdot 25 \cdot \cancel{24} \cdot 23 \cdot \cancel{22!}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{22!}}} 5^{22} 7^4 n^{56}$$

$$= \boxed{26 \cdot 25 \cdot 23} \boxed{5^{22}} \boxed{7^4} \boxed{n^{56}}$$

1 POINT EACH
☆ SUBTRACT 1 POINT IF YOUR ANSWER IS NEGATIVE

Use the entries of Pascal's Triangle to expand and simplify $(6z - 11t)^5$. SCORE: _____ / 5 PTS

NOTES: **Your coefficients may use the operations +, –, × and positive exponents only.**
All exponents must be simplified and all signs must be explicit (at the front).
You must show the intermediate step in the expansion to get full credit.

$$1(6z)^5(-11t)^0 + 5(6z)^4(-11t)^1 + 10(6z)^3(-11t)^2 + 10(6z)^2(-11t)^3 + 5(6z)^1(-11t)^4 + 1(6z)^0(-11t)^5$$

$$= \boxed{6^5 z^5} \boxed{- 5 \cdot 6^4 \cdot 11 z^4 t} \boxed{+ 10 \cdot 6^3 \cdot 11^2 z^3 t^2} \boxed{- 10 \cdot 6^2 \cdot 11^3 z^2 t^3} \boxed{+ 5 \cdot 6 \cdot 11^4 z t^4} \boxed{- 11^5 t^5}$$

½ POINT EACH ☆ SUBTRACT 1 POINT IF YOUR NEGATIVES IN THE FINAL ANSWER ARE INSIDE ( )'S.

Find the rational number representation of the repeating decimal $0.3\overline{45}$. SCORE: _____ / 5 PTS

NOTES: **Only the 45 is repeated.**
You must use only techniques from sections 9.2-9.5.

$\boxed{0.3 + 0.045 + 0.00045 + 0.0000045 + \dots}$ 1

$$= \frac{3}{10} + \boxed{\frac{\frac{45}{1000}}{1 - \frac{1}{100}}} \quad 1\tfrac{1}{2}$$

$$= \frac{3}{10} + \frac{\frac{45}{1000}}{\frac{99}{100}}$$

$$= \frac{3}{10} + \frac{\cancel{45}^{5}}{\cancel{1000}_{2}} \cdot \frac{\cancel{100}}{\cancel{99}_{11}}$$

½ $= \boxed{\frac{3}{10}} + \boxed{\frac{1}{22}}$ 1

$$= \frac{33+5}{110} = \frac{38}{110} = \boxed{\frac{19}{55}} \quad 1$$

AJ and BJ both have sequences where the third term is 81, and the sixth term is -24. **SCORE: _____ / 5 PTS**

[a] If AJ's sequence is geometric, find the first term of AJ's sequence.

$$\begin{aligned} a_3 &= \boxed{a_1 r^2 = 81} \\ a_6 &= \boxed{a_1 r^5 = -24} \end{aligned} \;\; \tfrac{1}{2} \longrightarrow \frac{a_1 r^5}{a_1 r^2} = \frac{-24}{81} \longrightarrow \boxed{r^3 = -\frac{8}{27}} \;\tfrac{1}{2} \longrightarrow \boxed{r = -\frac{2}{3}} \;\tfrac{1}{2}$$

$$a_1\left(-\tfrac{2}{3}\right)^2 = 81 \longrightarrow \tfrac{4}{9}a_1 = 81 \longrightarrow \boxed{a_1 = \frac{729}{4}} \;\tfrac{1}{2}$$

[b] If BJ's sequence is arithmetic, find the sum of the first 11 terms of BJ's sequence.

$$\begin{aligned} a_3 &= \boxed{a_1 + 2d = 81} \\ a_6 &= \boxed{a_1 + 5d = -24} \end{aligned} \;\; \tfrac{1}{2}$$

$$\boxed{-3d = 105} \;\tfrac{1}{2}$$

$$\boxed{d = -35} \;\tfrac{1}{2}$$

$$a_1 + 2(-35) = 81$$

$$a_1 = 151$$

$$S_{11} = \boxed{\tfrac{11}{2}(2(151) + (11-1)(-35))} \;1$$

$$= \tfrac{11}{2}(302 - 350) = \tfrac{11}{2} \cdot \cancel{-48}^{\,-24} = \boxed{-264} \;\tfrac{1}{2}$$

Using mathematical induction, prove that $\sum_{i=1}^{n}[(3i+1)\cdot 4^i] = n \cdot 4^{n+1}$ for all positive integers n. **SCORE: _____ / 10 PTS**

BASIS STEP: PROVE $\sum_{i=1}^{1}(3i+1)\cdot 4^i = 1\cdot 4^{1+1} = 1\cdot 4^2$

$$\sum_{i=1}^{1}(3i+1)\cdot 4^i = (3(1)+1)\cdot 4^1 = 4\cdot 4 = 16$$

$$1\cdot 4^2 = 16 \checkmark$$

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k}(3i+1)\cdot 4^i = k\cdot 4^{k+1}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

PROVE $\sum_{i=1}^{k+1}(3i+1)\cdot 4^i = (k+1)\cdot 4^{(k+1)+1} = (k+1)\cdot 4^{k+2}$

$$\begin{aligned} \sum_{i=1}^{k+1}(3i+1)4^i &= \sum_{i=1}^{k}(3i+1)\cdot 4^i + (3(k+1)+1)\cdot 4^{k+1} \\ &= k\cdot 4^{k+1} + (3k+4)\cdot 4^{k+1} \\ &= (k+3k+4)\cdot 4^{k+1} \\ &= (4k+4)4^{k+1} \\ &= 4(k+1)4^{k+1} \\ &= (k+1)4^{k+2} \end{aligned}$$

GRADED BY ME

BY MI, $\sum_{i=1}^{n}(3i+1)4^i = n\cdot 4^{n+1}$ FOR ALL POSITIVE INTEGERS n